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SELECTION OF A ZERO APPROXIMATION FOR THE POSITION OF AN ARTIFICIAL EARTH SATELLITE IN A PHASE OF ITS ORIENTED MOTION USING A DIPOLE APPROXIMATION OF THE GEOMAGNETIC FIELD

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16. Abstract

The problem of defining the zero approximation for the angular position of a round artificial Earth satellite of oriented motion is discussed using "aircraft" angles psi, phi and theta (yaw, bank, pitch, respectively). Trigonometric polynomials of the second power are used in terms of powers of the argument of latitude.

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SELECTION OF A ZERO APPROXIMATION FOR THE POSITION OF AN ARTIFICIAL EARTH SATELLITE IN A PHASE OF ITS ORIENTED MOTION USING A DIPOLE APPROXIMATION OF THE GEOMAGNETIC FIELD

V. S. Novoselov

1. Statement of the Problem /123

We will solve the problem of defining the zero approximation for the angular position of a round AES in the phase of oriented motion. In this phase, it is advisable to define the satellite's position using "aircraft" angles ψ, ϕ, and θ. Here ψ--the angle of yaw, ϕ--the angle of bank; and θ--the angle of pitch. For the oriented motion of a satellite, we may adopt an approximation of the angles ψ, ϕ and θ in the form of trigonometric polynomials in terms of powers of the argument of latitude. We will limit ourselves to considering terms of the second power:

$$\begin{aligned}\psi&=D_{11}+D_{12}\sin u+D_{13}\cos u+D_{14}\sin 2u+D_{15}\cos 2u,\\ \varphi&=D_{21}+D_{22}\sin u+D_{23}\cos u+D_{24}\sin 2u+D_{25}\cos 2u,\\ \theta&=D_{31}+D_{32}\sin u+D_{33}\cos u+D_{34}\sin 2u+D_{35}\cos 2u.\end{aligned} \tag{1}$$

In formula (1), D_{pq} = const (p = 1, 2, 3; q = 1, 2, 3, 4, 5) and u is the argument of satellite latitude. For the circular orbit in point $u = \omega_0 t$, where ω_0 is the orbital angular velocity, t is time measured from the time of passage across the equator.

As we know,the geomagnetic field can be approximated to within 12% by a dipole field with an axis coinciding with the Earth's axis of rotation. Thus in defining the angular position of the satellite, using a magnetometer to determin the zero approximation D^0_{pq}, we will proceed from a dipole model of the Earth's magnetic field.

Let us designate by H_x, H_y and H_z the projections of intensity of the geomagnetic field onto the axes of a system rigidly connected to the satellite. In complete orientation of the satellite, the x axis is given as directed along the positive transversal, the y axis--orthogonally to the orbital plane, and the z axis--along the radius-vector of the center of mass. We will use i to designate the orbital inclination of the AES to the equatorial plane, and H_0 will stand for some constant typical of the satellite's flight altitude.

On the basis of a dipole model for small angles of orienta- /124
tion, we will write

$$\left.\begin{aligned} H_0^{-1}H_x &= \sin i \cos u + \psi \cos i + 2\theta \sin i \sin u, \\ H_0^{-1}H_y &= -\psi \sin i \cos u + \cos i - 2\varphi \sin i \sin u, \\ H_0^{-1}H_z &= \theta \sin i \cos u - \varphi \cos i - 2 \sin i \sin u. \end{aligned}\right| \qquad (2)$$

Given that we know, by the readings of the magnetometer, the values H_x, H_y and H_z in some phase of oriented motion of the AES. To enhance the accuracy of these values, it is expedient to statistically smooth the tables for H_x, H_y and H_z. Let us establish the shape of approximating formulas. For this purpose, let us substitute the approximating expressions of (1) for the angles of orientation on the right sides of the relations of formula (2). We will then find that

$$\begin{gathered} H_0^{-1}H_x = D_{11} \cos i + D_{32} \sin i + \sin u \,(D_{12} \cos i + 2D_{31} \sin i - \\ - D_{35} \sin i) + \cos u \,(\sin i + D_{13} \cos i + D_{34} \sin i) + \sin 2u \,(D_{14} \cos i + \\ + D_{33} \sin i) + \cos 2u \,(D_{15} \cos i - D_{32} \sin i) + \\ + D_{35} \sin i \sin 3u - D_{34} \sin i \cos 3u, \end{gathered} \qquad (3)$$

$$H_0^{-1}H_y = -\frac{1}{2}D_{13}\sin i + \cos i - D_{22}\sin i + \sin u\left(-\frac{1}{2}D_{14}\sin i - -2D_{21}\sin i + D_{25}\sin i\right) + \cos u\left(-D_{11}\sin i - \frac{1}{2}D_{15}\sin i - D_{24}\sin i\right) + + \sin 2u\left(-\frac{1}{2}D_{12}\sin i - D_{23}\sin i\right) + + \cos 2u\left(-\frac{1}{2}D_{13}\sin i + D_{22}\sin i\right) + + \sin 3u\left(-\frac{1}{2}D_{14}\sin i - D_{25}\sin i\right) + + \cos 3u\left(-\frac{1}{2}D_{15}\sin i + D_{24}\sin i\right). \tag{4}$$

$$H_0^{-1}H_z = \frac{1}{2}D_{33}\sin i - D_{21}\cos i + \sin u\left(\frac{1}{2}D_{34}\sin i - 2\sin i - -D_{22}\cos i\right) + \cos u\left(D_{31}\sin i + \frac{1}{2}D_{35}\sin i - D_{23}\cos i\right) + + \sin 2u\left(\frac{1}{2}D_{32}\sin i - D_{24}\cos i\right) + + \cos 2u\left(\frac{1}{2}D_{33}\sin i - D_{25}\cos i\right) + \frac{1}{2}D_{34}\sin i\sin 3u + + \frac{1}{2}D_{35}\sin i\cos 3u. \tag{5}$$

Formulas (3)-(5) indicate that it is expedient to approximate the table of magnetometer readings in the form of trigonometric polynomials in terms of powers of the argument of latitude with the computation of terms to the third power exclusively: /125

$$H_x = H_{11} + H_{12}\sin u + H_{13}\cos u + H_{14}\sin 2u + + H_{15}\cos 2u + H_{16}\sin 3u + H_{17}\cos 3u, \tag{6}$$

$$H_y = H_{21} + H_{22}\sin u + H_{23}\cos u + H_{24}\sin 2u + H_{25}\cos 2u + + H_{26}\sin 3u + H_{27}\cos 3u, \tag{7}$$

$$H_z = H_{31} + H_{32}\sin u + H_{33}\cos u + H_{34}\sin 2u + + H_{35}\cos 2u + H_{36}\sin 3u + H_{37}\cos 3u. \tag{8}$$

In the zero approximation we will ignore errors in magnetometer readings after statistical smoothing. In other words, in the zero approximation we will equate the values H_x, H_y and H_z obtained through formulas (3)-(5) and (6)-(8). The coefficients D_{pq} will be marked with the exponent "zero."

Two approaches are possible. Approach (A), in which a statistical smoothing in terms of the complete formulas (6)-(8) is conducted, or the simplified approach (B), where statistical smoothing is done on truncated trigonometric polynomials which contain expansions to the second power, namely

$$H_x=\tilde{H}_{11}+\tilde{H}_{12}\sin u+\tilde{H}_{13}\cos u+\tilde{H}_{14}\sin 2u+\tilde{H}_{15}\cos 2u, \tag{9}$$

$$H_y=\tilde{H}_{21}+\tilde{H}_{22}\sin u+\tilde{H}_{23}\cos u+\tilde{H}_{24}\sin 2u+\tilde{H}_{25}\cos 2u, \tag{10}$$

$$H_z=\tilde{H}_{31}+\tilde{H}_{32}\sin u+\tilde{H}_{33}\cos u+\tilde{H}_{34}\sin 2u+\tilde{H}_{35}\cos 2u. \tag{11}$$

Let us state the following problems. Problem 1: is it possible, by using only magnetometer readings, to define D^0_{pq}. Since in section 2 it will be shown that problem 1 has a negative solution, problem 2 arises: how many additional conditions, which do not depend on the magnetometer readings, must we have to define D^0_{pq} and which formulas define D^0_{pq} if we use the value of angles ϕ, ψ and θ at several points in the orbit of AES motion. Problem 2 is solved in section 3. It is also advisable to state problem 3: to define D^0_{pq} if we know the readings of some second physical vector at several points in the orbit of satellite motion. This problem is examined in section 4 of this study.

2. Inadequacy of Magnetometer Readings for Selection of a Zero Approximation of the Angular Position of an Oriented AES using a Dipole Model of the Field

Let us examine the simplified approach, (B). Given magnetometer readings which have been statistically smoothed through formulas (9)-(11). We equate the free terms, and also the coefficients at sin u, cos u, sin 2u, cos 2u in formulas (3)-(5) and (9)-(11) respectively, multiplied in advance by H_0^{-1}. To define D_{pq}^0 we will find the following equations:

$$\begin{cases} D_{11}^0 \cos i + D_{32}^0 \sin i = \widetilde{H}_{11} H_0^{-1}, \\ D_{12}^0 \cos i + 2D_{31}^0 \sin i - D_{35}^0 \sin i = \widetilde{H}_{12} H_0^{-1}, \\ D_{13}^0 \cos i + D_{34}^0 \sin i = \widetilde{H}_{13} H_0^{-1} - \sin i, \\ D_{14}^0 \cos i + D_{33}^0 \sin i = \widetilde{H}_{14} H_0^{-1}, \\ D_{15}^0 \cos i - D_{32}^0 \sin i = \widetilde{H}_{15} H_0^{-1}; \end{cases} \tag{12}$$

$$\begin{cases} -\frac{1}{2} D_{13}^0 \sin i - D_{22}^0 \sin i = \widetilde{H}_{21} H_0^{-1} - \cos i, \\ -\frac{1}{2} D_{14}^0 \sin i - 2D_{21}^0 \sin i + D_{25}^0 \sin i = \widetilde{H}_{22} H_0^{-1}, \\ -D_{11}^0 \sin i - \frac{1}{2} D_{15}^0 \sin i - D_{24}^0 \sin i = \widetilde{H}_{23} H_0^{-1}, \\ -\frac{1}{2} D_{12}^0 \sin i - D_{23}^0 \sin i = \widetilde{H}_{24} H_0^{-1}, \\ -\frac{1}{2} D_{13}^0 \sin i + D_{22}^0 \sin i = \widetilde{H}_{25} H_0^{-1}; \end{cases} \tag{13}$$

$$
\begin{aligned}
&\tfrac{1}{2} D^0_{33} \sin i - D^0_{21} \cos i = \widetilde{H}_{31} H_0^{-1}, \\
&\tfrac{1}{2} D^0_{34} \sin i - D^0_{22} \cos i = \widetilde{H}_{32} H_0^{-1} + 2 \sin i, \\
&D^0_{31} \sin i + \tfrac{1}{2} D^0_{35} \sin i - D^0_{23} \cos i = \widetilde{H}_{33} H_0^{-1}, \\
&\tfrac{1}{2} D^0_{32} \sin i - D^0_{24} \cos i = \widetilde{H}_{34} H_0^{-1}, \\
&\tfrac{1}{2} D^0_{33} \sin i - D^0_{25} \cos i = \widetilde{H}_{35} H_0^{-1}.
\end{aligned}
\tag{14}
$$

Let us compose a matrix of coefficients for unknown D^0_{pq} in /127 equations (12)-(14)

$$
D_6 = \left\| \begin{matrix}
\cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i & 0 & 0 & 0 \\
0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sin i & 0 & 0 & 0 & \sin i \\
0 & 0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i & 0 \\
0 & 0 & 0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i & 0 & 0 \\
0 & 0 & 0 & 0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & -2\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 & 0 & 0 \\
-\sin i & 0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\sin i & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\sin i & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & \sin i & 0 & 0 & 0 & \frac{1}{2}\sin i \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & \frac{1}{2}\sin i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & \frac{1}{2}\sin i & 0 & 0
\end{matrix} \right\|
\tag{15}
$$

Given that $i \neq 0$ and $i \neq 180°$, i.e., the AES orbit is not equatorial. Let us multiply the elements of the 6th line of the matrix (15) by 2cotani, elements of the 12th line by -2 and add the obtained expressions with the corresponding elements of the corresponding elements of the 3rd line. Then the elements of the new third line will be zero. Given, furthermore, that $i \neq 90°$. Let us multiply the elements of the 4th line by 1/2 tan i, then elements of the 11th line by -2tan i, and the elements of the 15th line by tan i. We will add the obtained values with the corresponding elements of the 7th line. As a result, the elements of the new 7th line will be zero. Let us multiply the elements of the 1st line of the matrix D_B by tan i, elements of the fifth line by 1/2 tan i, elements of the 14th line by -tan i and the obtained expressions will be added to the corresponding elements of the 8th line. The elements of the new 8th line will be zero.

We have shown that the 3rd equation of system (12), and likewise the 2nd and 3rd equations of system (13) will depend on the other equations of (12)-(14). The rank of the matrix (15) is no greater than 12. Equations (12)-(14) are insufficient to define D^0_{pq}.

Let us examine the same problem using approach (A). In this case, statistical smoothing is done through formulas (6)-(8). We /128 equate free terms and also coefficients where sin u, cos u, sin 2u, sin 3 u, cos 3 u in formulas (3)-(5) and (6)-(8), respectively, have been previously multiplied by H_0^{-1}. To define D^0_{pq} we will have equations (12)-(14) in which the 'tilde' exponent above H_{pq} has been omitted, as well as the following additional equations:

$$D^0_{35}\sin i = H_{16}H_0^{-1}, \quad -D^0_{34}\sin i = H_{17}H_0^{-1}, \tag{16}$$

$$-\frac{1}{2}D^0_{14}\sin i - D^0_{25}\sin i = H_{26}H_0^{-1}, \tag{17}$$

$$-\frac{1}{2}D^0_{15}\sin i + D^0_{24}\sin i = H_{27}H_0^{-1}, \tag{18}$$

$$\frac{1}{2}D^0_{34}\sin i = H_{36}H_0^{-1}, \quad \frac{1}{2}D^0_{35}\sin i = H_{37}H_0^{-1}. \tag{19}$$

On the basis of equations (16) and (19) we can assume that

$$D^0_{34}=(H_0 \sin i)^{-1}\left(H_{36}-\frac{1}{2}H_{17}\right), \tag{20}$$

$$D^0_{35}=(H_0 \sin i)^{-1}\left(H_{37}+\frac{1}{2}H_{16}\right). \tag{21}$$

Equations (16) and (19) thereby will be excluded from the examination. By omitting dependents, the 3rd, 7th and 8th equations of system (12)-(14), we will have 14 equations of a truncated system (12)-(15) and (17), (18) to define 13 unknowns D^0_{pq} ($p = 1,2$; $q = 1, 2, 3, 4, 5$) and D^0_{31}, D^0_{32}, D^0_{33}.

Let us write a matrix of coefficients with unknowns

$$D_A = \left\| \begin{array}{ccccc ccccc ccc}
\cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i & 0 \\
0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sin i & 0 & 0 \\
0 & 0 & 0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i \\
0 & 0 & 0 & 0 & \cos i & 0 & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 \\
0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\sin i \\
0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & \sin i & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & \frac{1}{2}\sin i & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos i & 0 & 0 & \frac{1}{2}\sin i \\
0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 & 0
\end{array} \right\| \tag{22}$$

Let us consider, as before, the non-degenerate case $i \neq \{0°, 90°, 180°\}$. Let us multiply the elements of the 14th line of matrix D_A by 2 cotan i, elements of the 11th line by 2 and add to the corresponding elements of the 4th line. Then all elements of the new 4th line will be equal to zero. Let us multiply the /129 elements of the 2nd line of matrix (22) by $-\frac{1}{2}$, elements of the 6th line by -cotan i and add them to the corresponding elements of the 10th line. All elements of the new 10th line will be zero. Let us multiply the elements of the 4th line by 1/2, elements of the 14th line by cotan i. By adding the obtained expressions to the corresponding elements of the 11th line, we derive a new 11th line with zero elements. Let us now multiply the elements of the 3rd line of matrix D_A by $-\frac{1}{2}$, elements of the 13th line by -cotan i and add them to the elements of the 12th line. The elements of the new 12th line will all be equal to zero. We have shown that the 4th, 10th, 11th and 12th equations of this system will be functions of the other equations. The rank of matrix (22) is no greater than 10. Equations (12) and (17) are insufficient to define the desired 13 unknowns.

Let us now consider degenerate cases. Given that sin i = 0. The rank of matrix (15) is equal to 10. Equations (12) of approach (B) yield

$$D^0_{11}=H_{11}(\tilde{H}_0\cos i)^{-1},\quad D^0_{12}+\tilde{H}_{12}(H_0\cos i)^{-1}, \tag{23}$$

$D^0_{13}=\tilde{H}_{13}(H_0\cos i)^{-1}$, $D^0_{14}=\tilde{H}_{14}(H_0\cos i)^{-1}$, $D^0_{15}=\tilde{H}_{15}(H_0\cos i)^{-1}$. The left sides of equations (13) vanish. In this case we must have

$$\tilde{H}_{21}=H_0\cos i,\quad \tilde{H}_{22}=0,\quad \tilde{H}_{23}=0,\quad \tilde{H}_{24}=0,\quad \tilde{H}_{25}=0. \tag{24}$$

Equations (14) yield

$$\left.\begin{array}{c} D^0_{21}=-\tilde{H}_{31}(H_0\cos i)^{-1},\quad D^0_{22}=-\tilde{H}_{32}(H_0\cos i)^{-1},\\ D^0_{23}=-\tilde{H}_{33}(H_0\cos i)^{-1},\quad D^0_{24}=-\tilde{H}_{34}(H_0\cos i)^{-1},\\ D^0_{25}=-\tilde{H}_{35}(H_0\cos i)^{-1}. \end{array}\right| \tag{25}$$

Formulas (23)-(24) are derived according to formulas (2) where $\sin i = 0$.

Thus, for motion in the equatorial orbit, with the aid of a magnetometer in approach (B), we can uniquely define fluctuations in angles ψ and ϕ and pitch fluctuations of the satellite in the indicated plane are not defined. The zero approximation for the trigonometric approximation of satellite fluctuations in yaw and bank can be calculated through formulas (23) and (25). The left sides of additional equations (16)-(18), obtained by approach (A) for $\sin i = 0$, vanish. Thus in this case, approach (A) is equivalent to approach (B).

Let us consider a second degenerate case, $i = 90°$. The elementary transformations show that the rank of matrix (15) is equal to 8. We can use the first 4 equations of system (12) and (14) as our independents, and also equations of system (13). Thus to define the zero approximation of the satellite's angular position, appraoch (B) requires us to have another 6 independent conditions. For the particular case of plane oscillations of a satellite in polar orbit, the angle θ is subject to definition. Among equations of (12) only 4 independents will exist. And we require one additional independent condition. For the case $i = 90°$, equations (12) might be reduced to the form /130

$$\left.\begin{array}{c} 2D_{31}^{0}-D_{35}^{0}=\widetilde{H}_{12}H_{0}^{-1}, \quad D_{32}^{0}=\widetilde{H}_{11}H_{0}^{-1}=-\widetilde{H}_{15}H_{0}^{-1}, \\ D_{33}^{0}=\widetilde{H}_{14}H_{0}^{-1}, \\ D_{34}^{0}=-1+\widetilde{H}_{13}H_{0}^{-1}. \end{array}\right| \qquad (26)$$

Formulas (26) show that in this particular case, the coefficients D_{32}^{0}, D_{33}^{0} and D_{34}^{0} are defined by magnetometer readings uniquely; to define the coefficients D_{31}^{0} and D_{35}^{0}, however, we require additional independent condition.

We will discuss the second degenerate case with the aid of formulas of approach (A). Since equations (14) in this case depend on equations (12), from which only the first 4 are independents, then taking (17), (19) and (20) into account, we arrive at a system of equations for 13 unknowns D_{pq} ($p = 1, 2$; $q = 1, 2, 3, 4, 5$) and D^0_{31}, D^0_{32}, D^0_{33} with the following coefficient matrix:

$$D_A(i=90^\circ) = \left\| \begin{array}{ccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sin i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin i \\ 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & -2\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 \\ -\sin i & 0 & 0 & 0 & \frac{1}{2}\sin i & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & 0 & 0 & -\sin i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2}\sin i & 0 & 0 & 0 & \sin i & 0 & 0 & 0 & 0 \end{array} \right\| \qquad (27)$$

The coefficients D^0_{34} and D^0_{35} are defined by formulas (19) and (20). /131

The first, 2nd and 4th lines of matrix (27) show that in the given case the first three coefficients of the approximating trigonometric polynomial for angle θ will be uniquely defined. In reality, the 1st, 2nd and 4th equations of system (12), allowing for (20), here yield

$$\begin{gathered} D^0_{31} = \frac{1}{2}\left(H_{12} + \frac{1}{2}H_{16} + H_{37}\right)H_0^{-1}, \\ D^0_{32} = H_{11}H_0^{-1}, \quad D^0_{33} = H_{14}H_0^{-1}. \end{gathered} \qquad (28)$$

Thus, in approach (A), fluctuations in the plane of the polar orbit by magnetometer readings are uniquely defined. But in defining fluctuations in angles ψ and ϕ for 10 unknowns, we only have 7 equations. Thus we require 3 additional independent conditions.

3. Definition within the Framework of A Dipole Model of the Zero Approximation of the Angular Position of an Oriented Satellite, if the Values of the Angles for Several points are Known

Analysis of degenerate cases conducted in the preceeding section showed that several coefficients of approximating polynomials for orientation angles are defined by magnetometer readings uniquely, whereas to define other coefficients additional independent conditions are required. It appears useful for undegenerate cases to isolate those coefficients of approximating polynomials which uniquely are defined by magnetometer readings, to find clear expressiones for these coefficients, and then examine the question of defining the remaining coefficients in the presence of additional information on the value of orientation angles at several points in the satellite's orbit.

We will consider a undegenerate case, i.e., given that sin i ≠ 0, cos i ≠ 0, the 3rd, 7th and 8th equations of system (12)-(14) are dropped, since they are dependent on the others. The remaining 12 equations of system (12)-(14) are transformed with the aid of elementary transformations. Equations of system (12)-(14) will be written thus:

$$D^0_{11}\cos i + D^0_{32}\sin i = \widetilde{H}_{11}H_0^{-1}, \tag{29}$$

$$D^0_{35} = \left(-\frac{1}{2}\widetilde{H}_{12} - \operatorname{ctg} i\,\widetilde{H}_{24} + \widetilde{H}_{33}\right)(H_0 \sin i)^{-1}, \tag{30}$$

$$D^0_{14}\cos i + D^0_{33}\sin i = \widetilde{H}_{14}H_0^{-1}, \tag{31}$$

$$D^0_{11} + D^0_{15} = (\widetilde{H}_{11} + \widetilde{H}_{15})(H_0 \cos i)^{-1}, \tag{32}$$

$$D^0_{22} = \frac{1}{2}\operatorname{ctg} i - \frac{1}{2}(\widetilde{H}_{21} - \widetilde{H}_{25})(H_0 \sin i)^{-1}, \tag{33}$$

$$\frac{1}{2}D^0_{12}+D^0_{23}=-\widetilde{H}_{24}(H_0\sin i)^{-1}, \tag{34}$$ /132

$$D^0_{13}=\operatorname{ctg} i-(\widetilde{H}_{21}+\widetilde{H}_{25})(H_0\sin i)^{-1}, \tag{35}$$

$$D^0_{14}+2D^0_{21}=(\widetilde{H}_{14}-2\widetilde{H}_{31})(H_0\cos i)^{-1}, \tag{36}$$

$$D^0_{34}=4+\operatorname{ctg}^2 i-(\operatorname{ctg} i\,\widetilde{H}_{21}-\operatorname{ctg} i\,\widetilde{H}_{25}-2\widetilde{H}_{32})(H_0\sin i)^{-1}, \tag{37}$$

$$D^0_{23}\cos i-D^0_{31}\sin i=-\frac{1}{2}\left(\frac{1}{2}\widetilde{H}_{12}+\operatorname{ctg} i\,\widetilde{H}_{24}+\widetilde{H}_{33}\right)H_0^{-1}, \tag{38}$$

$$D^0_{11}+2D^0_{24}=(\widetilde{H}_{11}-2\widetilde{H}_{34})(H_0\cos i)^{-1}, \tag{39}$$

$$D^0_{21}-D^0_{25}=-(\widetilde{H}_{31}-\widetilde{H}_{35})(H_0\cos i)^{-1}. \tag{40}$$

The system of equations (29)-(40) shows that 4 coefficients D^0_{13}, D^0_{22}, D^0_{34} and D^0_{35} are defined uniquely by magnetometer readings. The other 11 unknowns have 8 equations (29), (31), (32), (34), (36), (38-40).

Given that the values of angles ψ, ϕ, and θ are defined for values of the argument of latitude u_k with the aid of local control of orientation by measurement of two or more physical vectors. In this case, additional equations (which follow) can be composed:

$$D^0_{11}+D^0_{12}\sin u_k+D^0_{13}\cos u_k+D^0_{14}\sin 2u_k+D^0_{15}\cos 2u_k=\psi(u_k), \tag{41}$$

$$D^0_{21}+D^0_{22}\sin u_k+D^0_{23}\cos u_k+D^0_{24}\sin 2u_k+D^0_{25}\cos 2u_k=\varphi(u_k), \tag{42}$$

$$D^0_{31}+D^0_{32}\sin u_k+D^0_{33}\cos u_k+D^0_{34}\sin 2u_k+D^0_{35}\cos 2u_k=\theta(u_k). \tag{43}$$

With the aid of formulas (30), (33), (35), (37), we will exclude from equations (41)-(43) the coefficients D^0_{13}, D^0_{22}, D^0_{34}, D^0_{35}; then we transform the obtained equations, using formulas (29), (31), (34), (36), (38-40). The sequence of transformations is reflected on the right sides of the relationships which follow--these are transformed equations for (41)-(43), respectively:

$$D_{11}^0 \sin u_k + D_{14}^0 \cos u_k - D_{23}^0 = \frac{1}{2\sin u_k}\Big[\psi(u_k) - D_{13}^0 \cos u_k - \\ - \frac{\cos 2u_k}{\cos i}(\widetilde{H}_{11} + \widetilde{H}_{15}) H_0^{-1} + 2\frac{\sin u_k}{\sin i}\widetilde{H}_{24} H_0^{-1}\Big] = \Psi_k, \tag{44}$$

$$-(D_{11}^0 \sin u_k + D_{14}^0 \cos u_k) + D_{23}^0 = \frac{1}{\cos u_k}\Big[\varphi(u_k) - D_{22}^0 \sin u_k + \\ + \frac{\cos 2u_k}{\cos i}(\widetilde{H}_{35} - \widetilde{H}_{31}) H_0^{-1} - \frac{1+\cos 2u_k}{\cos i}\left(\frac{1}{2}\widetilde{H}_{14} - \widetilde{H}_{31}\right) H_0^{-1} - \\ - \frac{\sin 2u_k}{\cos i}\left(\frac{1}{2}\widetilde{H}_{11} - \widetilde{H}_{34}\right) H_0^{-1}\Big] = -\Phi_k, \tag{45}$$

$$-(D_{11}^0 \sin u_k + D_{14}^0 \cos u_k) + D_{23}^0 = \operatorname{tg} i\Big[\theta(u_k) - D_{34}^0 \sin 2u_k - \\ - D_{35}^0 \cos 2u_k - \frac{\sin u_k}{\sin i}\widetilde{H}_{11} H_0^{-1} - \frac{\cos u_k}{\sin i}\widetilde{H}_{14} H_0^{-1} - \\ - \frac{1}{2\sin i}\left(\frac{1}{2}\widetilde{H}_{12} + \operatorname{ctg} i\,\widetilde{H}_{24} + \widetilde{H}_{33}\right) H_0^{-1}\Big] = -\Theta_k. \tag{46}$$

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In writing equations (44)-(46), cases of $\sin u_k = 0$ and $\cos u_k = 0$ are eliminated. The left sides of equations (45) and (46) are derived from the left side of equation (44) by multiplying it by -1. System (44)-(46) contains only one independent equation.

Let us introduce the notation

$$Q_k = \frac{1}{3}(\Psi_k + \Phi_k + \Theta_k). \tag{47}$$

On the average, system (44)-(46) is tantamount to a single equation

$$D_{11}^0 \sin u_k + D_{14}^0 \cos u_k - D_{23}^0 = Q_k. \tag{48}$$

For the completeness of the system of determinant equations, we must minimally know the angular position of the AES at three different points for u_1, u_2 and u_3. Let us fulfill the sequential computations of equations of type (48) for the three indicated values of the argument of latitude

$$\begin{aligned} D_{11}^0 (\sin u_1 - \sin u_2) + D_{14}^0 (\cos u_1 - \cos u_2) = Q_1 - Q_2, \\ D_{11}^0 (\sin u_1 - \sin u_3) + D_{14}^0 (\cos u_1 - \cos u_3) = Q_1 - Q_3. \end{aligned} \tag{49}$$

We will resolve equations (49) relative to D_{11}^0 and D_{14}^0

$$D_{11}^0 = \frac{Q_1 (\cos u_3 - \cos u_2) + Q_2 (\cos u_1 - \cos u_3) + Q_3 (\cos u_2 - \cos u_1)}{\sin (u_3 - u_2) + \sin (u_1 - u_3) + \sin (u_2 - u_1)}, \tag{50}$$

$$D_{14}^0 = -\frac{Q_1 (\sin u_3 - \sin u_2) + Q_2 (\sin u_1 - \sin u_3) + Q_3 (\sin u_2 - \sin u_1)}{\sin (u_3 - u_2) + \sin (u_1 - u_3) + \sin (u_2 - u_1)}. \tag{51}$$

With the aid of equation (48) we will find, on the average, that

$$\begin{aligned} D_{23}^0 = \frac{1}{3} \Big[D_{11}^0 (\sin u_1 + \sin u_2 + \sin u_3) + \\ + D_{14}^0 (\cos u_1 + \cos u_2 + \cos u_3) - Q_1 - Q_2 - Q_3 \Big]. \end{aligned} \tag{52}$$

The clear expressions found above were for 7 coefficients D_{11}^0, D_{13}^0, D_{14}^0, D_{22}^0, D_{23}^0, D_{34}^0 and D_{35}^0. Equations (32) and (34) make it possible to define D_{12}^0 and D_{15}^0 in the form

$$D_{12}^0 = -2 (\widetilde{H}_{24} H_0^{-1} \sin i^{-1} + D_{23}^0), \tag{53}$$

$$D_{15}^0 = (\widetilde{H}_{11} + \widetilde{H}_{16}) (H_0 \cos i)^{-1} - D_{11}^0. \tag{54}$$

With the aid of formulas (36), (39), and (40) we find that /134

$$D_{21}^0 = \left(\frac{1}{2} \widetilde{H}_{14} - \widetilde{H}_{31}\right) (H_0 \cos i)^{-1} - \frac{1}{2} D_{14}^0, \tag{55}$$

$$D_{24}^0 = \left(\frac{1}{2} \widetilde{H}_{11} - \widetilde{H}_{34}\right) (H_0 \cos i)^{-1} - \frac{1}{2} D_{11}^0, \tag{56}$$

$$D^0_{25}=(\widetilde{H}_{31}-\widetilde{H}_{35})(H_0\cos i)^{-1}+D^0_{12}. \tag{57}$$

Equations (29), (31) and (38) permit us to define the remaining unknowns

$$D^0_{31}=\frac{1}{2}\left(\frac{1}{2}\widetilde{H}_{12}+\operatorname{ctg} i\,\widetilde{H}_{24}+\widetilde{H}_{33}\right)+(H_0\sin i)^{-1}+D^0_{23}\operatorname{ctg} i, \tag{58}$$

$$D^0_{32}=\widetilde{H}_{11}(H_0\sin i)^{-1}-D^0_{11}\operatorname{ctg} i, \tag{59}$$

$$D^0_{33}=\widetilde{H}_{14}(H_0\sin i)^{-1}-D^0_{14}\operatorname{ctg} i. \tag{60}$$

We have examined this case for approach (B). Let us find out what simplifications can be derived by using approach (A). In approach (A) we also need 3 additional independent equations; we will retain equations (29)-(40), but on the right sides we drop the exponent 'tilde.' Coefficients D^0_{34} and D^0_{35} can be calculated both by formulas (20), (21) and by formulas (30), (37). The use of data on the angular position at two points is done just as in approach (B). We will have the same final formulas (48), (51), and (52). Furthermore, formulas (53)-(60) can be used, but in the first parts of these we omit the exponent 'tilde.' The coefficients D^0_{24} and D^0_{25} can also be defined by formulas obtainable from (17)-(18)

$$D^0_{24}=H_{27}(H_0\sin i)^{-1}+\frac{1}{2}D^0_{15}, \tag{61}$$

$$D^0_{25}=-H_{26}(H_0\sin i)^{-1}-\frac{1}{2}D^0_{14}. \tag{62}$$

Thus, the use of formulas of approach (A) does not lead to any simplifications. We should note that to calculate by formulas (20), (21) and (61), (62) which take place only for approach (A) is hardly expedient, since these formulas contain less precisely defined coefficients with third harmonics. Thus the working formulas will be (30), (33), (35), (37), (48), (51)-(60), which are valid for approaches (A) and (B). But the statistical smoothing of measured values of H_x, H_y and H_z is best done by formulas (6)-(8), since more reliable values of the coefficients will be obtained.

4. Definition in a Dipole Model of the Zero Approximation of the Angular Position of an Oriented Satellite if the Direction of the Second Vector is Known for Several Points

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Given in a system of coordinates (x, y, z) rigidly connected to the body of the AES we make a measurement of the direction of some vector S (direction toward the Sun, Earth or other body). For a point with the argument of latitude u_k we will have measured values cos (S_k, x), cos (S_k, y) and cos(S_k, z). According to the model of motion of the observable body directional cosines of the same direction have been calculated in the orbital system: cos(S_k, x_0), cos(S_k, y_0) and cos(S_k, z_0). The x_0, y_0, z_0 axes coincide with the x, y, z axes in satellite orientation.

For small angles of orientation, by analogy with formula (2) we will write

$$\begin{aligned}\cos(S_k, x)&=\cos(S_k, x_0)+\psi\cos(S_k, y_0)-\theta\cos(S_k, z_0),\\ \cos(S_k, y)&=-\psi\cos(S_k, x_0)+\cos(S_k, y_0)+\varphi\cos(S_k, z_0),\\ \cos(S_k, z)&=\theta\cos(S_k, x_0)-\varphi\cos(S_k, y_0)+\cos(S_k, z_0).\end{aligned} \tag{63}$$

Let us substitute expressions (1) in formula (63)

$$\begin{aligned}&(D^0_{11}+D^0_{12}\sin u_k+D^0_{13}\cos u_k+D^0_{14}\sin 2u_k+D^0_{15}\cos 2u_k)\cos(S_k, y_0)-\\ &-(D^0_{31}+D^0_{32}\sin u_k+D^0_{33}\cos u_k+D^0_{34}\sin 2u_k+D^0_{35}\cos 2u_k)\cos(S_k, z_0)=\\ &=\cos(S_k, x)-\cos(S_k, x_0),\end{aligned} \tag{64}$$

$$-(D_{11}^0+D_{12}^0\sin u_k+D_{13}^0\cos u_k+D_{14}\sin 2u_k+D_{15}^0\cos 2u_k)\cos(S_k,x_0)+$$
$$+(D_{21}^0+D_{22}^0\sin u_k+D_{23}^0\cos u_k+D_{24}^0\sin 2u_k+D_{25}^0\cos 2u_k)\cos(S_k,z_0)=$$
$$=\cos(S_k,y)-\cos(S_k,y_0), \qquad (65)$$

$$-(D_{21}^0+D_{22}^0\sin u_k+D_{23}^0\cos u_k+D_{24}^0\sin 2u_k+D_{25}^0\cos 2u_k)\cos(S_k,y_0)+$$
$$+(D_{31}^0+D_{32}^0\sin u_k+D_{33}^0\cos u_k+D_{34}^0\sin 2u_k+D_{35}^0\cos 2u_k)\cos(S_k,x_0)=$$
$$=\cos(S_k,z)-\cos(S_k,z_0). \qquad (66)$$

With the aid of formulas (30), (33), (35), (37) we will exclude from equations (64)-(66) the coefficients D_{13}^0, D_{22}^0, D_{34}^0, D_{35}^0; then transform the obtained equations, using formulas (29), (31), (34), (36), (38)-(40). The sequence of transformations is reflected on the right sides of the transformations cited below

$$D_{11}^0\sin u_k+D_{14}^0\cos u_k-D_{23}^0=[2\sin u_k\cos(S_k,y_0)+$$
$$+\operatorname{ctg} i\cos(S_k,z_0)]^{-1}[\cos(S_k,x)-\cos(S_k,x_0)-$$
$$-D_{13}^0\cos u_k\cos(S_k,y_0)+D_{34}^0\sin 2u_k\cos(S_k,z_0)+$$
$$+D_{35}^0\cos 2u_k\cos(S_k,z_0)+\widetilde{H}_{11}(H_0\sin i)^{-1}\sin u_k\cos(S_k,z_0)+$$
$$+\widetilde{H}_{14}(H_0\sin i)^{-1}\cos u_k\cos(S_k,z_0)+\frac{1}{2}\left(\frac{1}{2}\widetilde{H}_{12}+\operatorname{ctg} i\,\widetilde{H}_{24}+\widetilde{H}_{33}\right)\times$$
$$\times(H_0\sin i)^{-1}\cos(S_k,z_0)-(\widetilde{H}_{11}+\widetilde{H}_{15})(H_0\cos i)^{-1}\cos 2u_k\cos(S_k,y_0)+$$
$$+2H_{24}(\widetilde{H}_0\sin i)^{-1}\sin u_k\cos(S_k,y_0)]=\Psi'_k, \qquad (67)$$

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$$-(D_{11}^0\sin u_k+D_{14}^0\cos u_k)+D_{23}^0=$$
$$=[2\sin u_k\cos(S_k,x_0)+\cos u_k\cos(S_k,z_0)]^{-1}[\cos(S_k,y)-\cos(S_k,y_0)+$$
$$+D_{13}^0\cos u_k\cos(S_k,x_0)-D_{22}^0\sin u_k\cos(S_k,z_0)+$$
$$+(\widetilde{H}_{35}-\widetilde{H}_{31})(H_0\cos i)^{-1}\cos 2u_k\cos(S_k,z_0)-\left(\frac{1}{2}\widetilde{H}_{11}-\widetilde{H}_{34}\right)\times$$
$$\times(H_0\cos i)^{-1}\sin 2u_k\cos(S_k,z_0)-2\left(\frac{1}{2}\widetilde{H}_{14}-\widetilde{H}_{31}\right)\times$$
$$\times(H_0\cos i)^{-1}\cos^2 u_k\cos(S_k,z_0)+(\widetilde{H}_{11}+\widetilde{H}_{15})(H_0\cos i)^{-1}\times$$
$$\times\cos 2u_k\cos(S_k,x_0)-2\widetilde{H}_{24}(H_0\sin i)^{-1}\sin u_k\cos(S_k,x_0)]=-\Phi'_k, \qquad (68)$$

$$
\begin{aligned}
&-(D_{11}^{0}\sin u_k+D_{14}^{0}\cos u_k)+D_{23}^{0}=\\
&=[-\cos u_k\cos(S_k,\ y_0)+\operatorname{ctg} i\cos(S_k,\ x_0)]^{-1}[\cos(S_k,\ z)-\cos(S_k,\ z_0)+\\
&+D_{22}^{0}\sin u_k\cos(S_k,\ y_0)-(D_{34}^{0}\sin 2u_k+D_{35}^{0}\cos 2u_k)\cos(S_k,\ x_0)-\\
&-\sin^{-1} i\sin u_k\cos(S_k,\ x_0)\widetilde{H}_{11}H_0^{-1}-\sin^{-1} i\cos u_k\cos(S_k x_0)\widetilde{H}_{14}H_0^{-1}-\\
&-\frac{1}{2}\sin^{-1} i\cos(S_k,\ x_0)\left(\frac{1}{2}\widetilde{H}_{12}+\operatorname{ctg} i\,\widetilde{H}_{24}+\widetilde{H}_{33}\right)H_0^{-1}+\\
&+2\cos^{-1} i\cos^2 u_k\cos(S_k,\ y_0)\left(\frac{1}{2}\widetilde{H}_{14}-\widetilde{H}_{31}\right)H_0^{-1}+\\
&+\cos^{-1} i\sin 2u_k\cos(S_k,\ y_0)\left(\frac{1}{2}\widetilde{H}_{11}-\widetilde{H}_{34}\right)H_0^{-1}+\\
&+\cos^{-1} i\cos 2u_k\cos(S_k,\ y_0)(\widetilde{H}_{31}-\widetilde{H}_{35})H_0^{-1}]=-\Theta'_k.
\end{aligned}
\tag{69}
$$

Equations (67)÷(69) are a system of 3 equations of which only one is independent. For the correct definition of the zero approximation we must know the direction of the second physical vector at at least 3 different points in the orbit.

Equations (67)-(69) have the form of equations (44)-(46), if in the latter the quantities Ψ_k, Φ_k and Θ_k are replaced by Ψ'_k, Φ'_k and Θ'_k, respectively. Thus to define D_{11}^0, D_{14}^0 and D_{23}^0 we have formulas (50)-(52), in which Q_k is replaced by the quantities

$$Q'_k=\frac{1}{3}(\Psi'_k+\Phi'_k+\Theta'_k). \tag{70}$$

5. General Remarks on the Method of Spectral Approximation

1. Equations of system (2) are functionally dependent, since $\frac{D(H_x,\ H_y,\ H_z)}{D(\psi,\phi,\ \theta)}=0$. The corresponding relationship can be written as

$$\operatorname{ctg} i\,H_y=\sin i\left(\frac{5}{2}+\operatorname{ctg}^2 i-\frac{3}{2}\cos 2u\right)H_0-H_x\cos u+2H_z\sin u. \tag{71}$$

Thus, omitting equations (13), we come to 10 equations (12) and (14), the rank of matrix for which in the undegenerate case is 10. /137

This derviation follows from the fact that the determinant for unknown $D_{\theta q}$ and D_2^0 is equal to $(-1)^5 \cos^{10} i$. Calculation of terms of the third power in approach (A) will permit us to write additional equations (16) or (19). We find, as in section 2, that we have a system with rank 12.

In approach (B) we have no additional equations (16), (19). But the equations derived in section 2, as research in sections 3 and 4 showed, also have rank 12. To explain the cause of this circumstance, we equate free terms, and also terms where sin u, cos u, sin 2u and cos 2u are on the left and right sides of formula (71). We will have, respectively

$$\left.\begin{aligned}
&\operatorname{ctg} i\, H_{21} = \sin i \left(\frac{5}{2} + \operatorname{ctg}^2 i\right) H_0 - \frac{1}{2} H_{13} + H_{32},\\
&\operatorname{ctg} i\, H_{22} = -\frac{1}{2} H_{14} + 2H_{31} - H_{35},\\
&\operatorname{ctg} i\, H_{23} = -H_{11} - \frac{1}{2} H_{15} + H_{34},\\
&\operatorname{ctg} i\, H_{24} = -\frac{1}{2} H_{12} + H_{33} - \frac{1}{2} H_{16} - H_{37},\\
&\operatorname{ctg} i\, H_{25} = -\frac{1}{2} H_{13} - H_{32} - \frac{1}{2} H_{17} + H_{36}.
\end{aligned}\right| \qquad (72)$$

Since two last equations of system (72) contain coefficients in trigonometric functions sin 3u and cos 3u in approximating formulas for H_x and H_z (not examined in approach B), then these two equations in this approach should not be satisfied. Thus in approach (B) relationship (71) must be satisfied only partially ; it becomes possible to derive two independent equations. For the problem of selecting the zero approximation which will then be refined, this position is completely admissible.

2. As was noted, system (2) has only two functionally

independent equations. The question arises as to whether or not this system can be supplemented after differentiated equations (2)

$$\begin{aligned} H_0^{-1}H'_x &= -\sin i \sin u + \psi' \cos i + 2\theta' \sin i \sin u + 2\theta \sin i \cos u, \\ H_0^{-1}H'_y &= -\psi' \sin i \cos u + \psi \sin i \sin u - 2\varphi' \sin i \sin u - \\ &\quad - 2\varphi \sin i \cos u, \\ H_0^{-1}H'_z &= \theta' \sin i \cos u - \theta \sin i \sin u - \varphi' \cos i + 2 \sin i \cos u. \end{aligned} \tag{73}$$

If the approximating formulas (1) are substituted in (73), and the coefficients in sin u, cos u, sin 2u, cos 2u on the left and right sides of equation (73) are equated, we derive 12 equations to define 15 unknown D^0_{pq}. But these equations will coincide with /138 the system consisting of the last 4 equations of each of systems (12-(14), since the operation of differentiation and substitution instead of ψ, ϕ and θ of the approximating polynomials can exchange places. Thus in the use of spectral approximation of the orientation angles for some interval of time of oscillatory motion of the AES about the center of mass, the differentiated relations in (2) are automatically satisfied.

3. It was suggested above that the axis of the magnetic dipole coincides with the Earth's axis of rotation. If we reject this suggestion, then the displacement of the magnetic pole both in latitude and longitude will be calculated by the formulas

$$\begin{aligned} H_0^{-1}H_x &= \sin i_M \cos(u+u_0) + \psi \cos i_M + 2\theta \sin i_M \sin(u+u_0), \\ H_0^{-1}H_z &= -\psi \sin i_M \cos(u+u_0) + \cos i_M - 2\varphi \sin i_M \sin(u+u_0), \\ H_0^{-1}H_z &= \theta \sin i_M \cos(u+u_0) - \varphi \cos i_M - 2 \sin i_M \sin(u+u_0). \end{aligned} \tag{74}$$

Here i_M is the inclination of the plane of the circular orbit of the center of mass of the satellite to the magnetic equator; u_0 is some constant. Expressions (2) and (74) show that all formulas derived in this study will take place for the case in point, if the arguments of latitude u are replaced by $u + u_0$, and inclination i by i_M. The general conclusions on the methods of constructing the zero approximation for the angular position of an artificial satellite in the phase of oriented motion remain in force --when the more precise model of the geomagnetic field is used.

4. To enhance the accuracy of the zero approximation, it is useful to define the geomagnetic field model with the aid of a fragment of the Gaussian series. The methods of the present study can be used if projections of intensity of the geomagnetic field onto the axes of the orbital system of coordinates are smoothed in advance in the form of trigonometric polynomials in terms of powers of the argument of latitude.